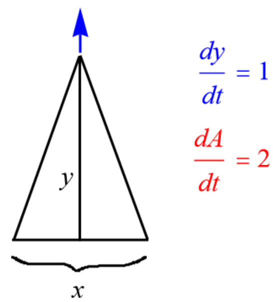
Exercise 21

The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2?

Solution

Draw a schematic of the triangle at some time.



The area of this triangle is

$$A = \frac{1}{2}xy. \tag{1}$$

Take the derivative of both sides with respect to time by using the chain rule.

$$\frac{d}{dt}(A) = \frac{d}{dt}\left(\frac{1}{2}xy\right)$$

$$\frac{dA}{dt} = \frac{1}{2}\left(\frac{dx}{dt}y + x\frac{dy}{dt}\right)$$

$$2 = \frac{1}{2}\left(\frac{dx}{dt}y + x \cdot 1\right)$$

$$4 = \frac{dx}{dt}y + x$$

Solve for dx/dt, the rate of change of the base with respect to time.

$$\frac{dx}{dt} = \frac{4 - x}{y}$$

When the altitude is 10 cm and the area is 100 cm^2, the necessary value of x is found from equation (1).

$$A = \frac{1}{2}xy \quad \rightarrow \quad x = \frac{2A}{y} = \frac{2(100)}{10} = 20$$

Therefore, when the altitude is 10 cm and the area is 100 cm^2, the rate that the base is changing is

$$\left.\frac{dx}{dt}\right|_{\substack{x=20\\y=10}} = \frac{4-(20)}{(10)} = -1.6\ \frac{\text{cm}}{\text{min}}.$$